

Paper Reference(s)

**6666/01**

# **Edexcel GCE**

## **Core Mathematics C4**

### **Bronze Level B2**

**Time: 1 hour 30 minutes**

**Materials required for examination**  
**papers**

Mathematical Formulae (Green)

**Items included with question**

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

#### **Instructions to Candidates**

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Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

#### **Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 8 questions in this question paper. The total mark for this paper is 75.

#### **Advice to Candidates**

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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

#### **Suggested grade boundaries for this paper:**

<b>A*</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>70</b>	<b>63</b>	<b>57</b>	<b>53</b>	<b>48</b>	<b>42</b>

1. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3},$$

find the binomial expansion of  $f(x)$ , in ascending powers of  $x$ , up to and including the term in  $x^3$ .

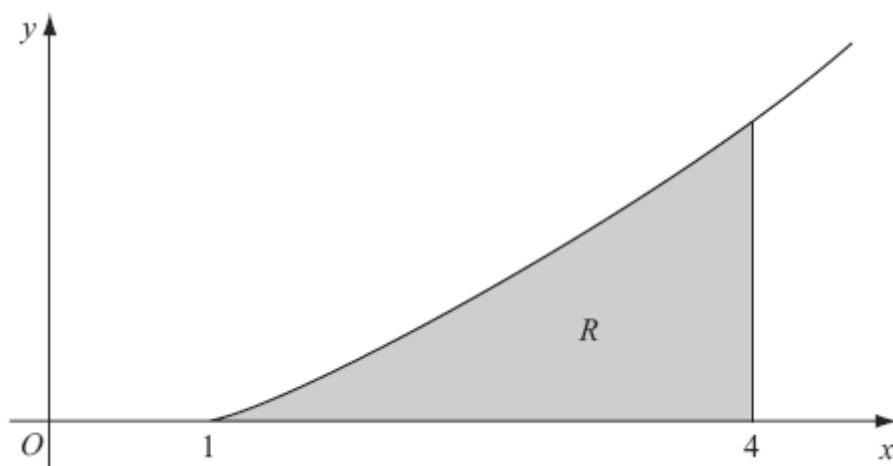
Give each coefficient as a simplified fraction.

**(5)**

**January 2013**

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2.



**Figure 1**

Figure 1 shows a sketch of the curve with equation  $y = x \ln x$ ,  $x \geq 1$ . The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis and the line  $x = 4$ .

The table shows corresponding values of  $x$  and  $y$  for  $y = x \ln x$ .

$x$	1	1.5	2	2.5	3	3.5	4
$y$	0	0.608			3.296	4.385	5.545

(a) Copy and complete the table with the values of  $y$  corresponding to  $x = 2$  and  $x = 2.5$ , giving your answers to 3 decimal places.

**(2)**

(b) Use the trapezium rule, with all the values of  $y$  in the completed table, to obtain an estimate for the area of  $R$ , giving your answer to 2 decimal places.

**(4)**

(c) (i) Use integration by parts to find  $\int x \ln x \, dx$ .

(ii) Hence find the exact area of  $R$ , giving your answer in the form  $\frac{1}{4}(a \ln 2 + b)$ , where  $a$  and  $b$  are integers.

**(7)**

**January 2010**

3. 
$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}.$$

(a) Find the values of the constants  $A$ ,  $B$  and  $C$ .

(4)

(b) (i) Hence find  $\int f(x) \, dx$ .

(3)

(ii) Find  $\int_0^2 f(x) \, dx$  in the form  $\ln k$ , where  $k$  is a constant.

(3)

June 2009

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4. 
$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(a) Find the values of the constants  $A$ ,  $B$  and  $C$ .

(4)

(b) Hence show that the exact value of  $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)} \, dx$  is  $2 + \ln k$ , giving the value of the constant  $k$ .

(6)

June 2007

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5. (a) Expand  $\frac{1}{\sqrt{4-3x}}$ , where  $|x| < \frac{4}{3}$ , in ascending powers of  $x$  up to and including the term in  $x^2$ . Simplify each term.

(5)

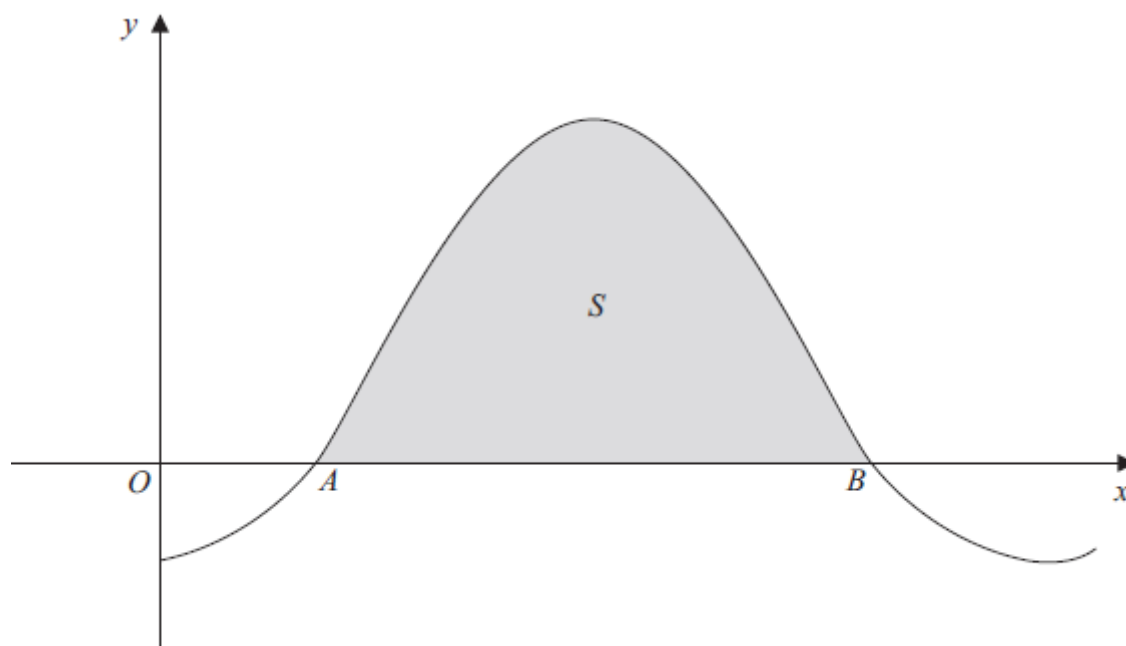
(b) Hence, or otherwise, find the first 3 terms in the expansion of  $\frac{x+8}{\sqrt{4-3x}}$  as a series in ascending powers of  $x$ .

(4)

June 2008

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6.



**Figure 3**

Figure 3 shows a sketch of part of the curve with equation  $y = 1 - 2 \cos x$ , where  $x$  is measured in radians. The curve crosses the  $x$ -axis at the point  $A$  and at the point  $B$ .

- (a) Find, in terms of  $\pi$ , the  $x$  coordinate of the point  $A$  and the  $x$  coordinate of the point  $B$ . (3)

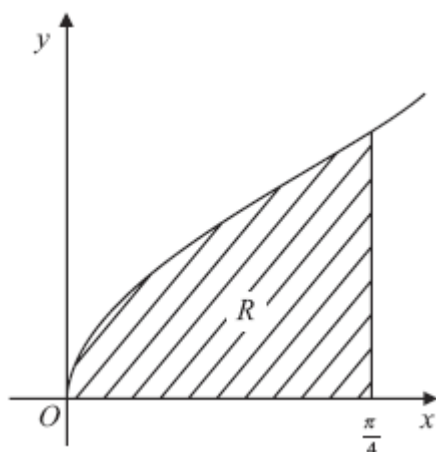
The finite region  $S$  enclosed by the curve and the  $x$ -axis is shown shaded in Figure 3. The region  $S$  is rotated through  $2\pi$  radians about the  $x$ -axis.

- (b) Find, by integration, the exact value of the volume of the solid generated. (6)

**January 2013**

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7.



**Figure 1**

Figure 1 shows part of the curve with equation  $y = \sqrt[3]{(\tan x)}$ . The finite region  $R$ , which is bounded by the curve, the  $x$ -axis and the line  $x = \frac{\pi}{4}$ , is shown shaded in Figure 1.

- (a) Given that  $y = \sqrt[3]{(\tan x)}$ , copy and complete the table with the values of  $y$  corresponding to  $x = \frac{\pi}{16}$ ,  $\frac{\pi}{8}$  and  $\frac{3\pi}{16}$ , giving your answers to 5 decimal places.

$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
$y$	0				1

(3)

- (b) Use the trapezium rule with all the values of  $y$  in the completed table to obtain an estimate for the area of the shaded region  $R$ , giving your answer to 4 decimal places.

(4)

The region  $R$  is rotated through  $2\pi$  radians around the  $x$ -axis to generate a solid of revolution.

- (c) Use integration to find an exact value for the volume of the solid generated.

(4)

**June 2007**

8. (a) Find  $\int (4y+3)^{-\frac{1}{2}} dy$ .

(2)

(b) Given that  $y=1.5$  at  $x=-2$ , solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{4y+3}}{x^2},$$

giving your answer in the form  $y = f(x)$ .

(6)

June 2011

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**TOTAL FOR PAPER: 75 MARKS**

**END**

Question Number	Scheme	Marks
1.	$(2+3x)^{-3} = \underline{(2)}^{-3} \left(1 + \frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}} \left(1 + \frac{3x}{2}\right)^{-3}$ $= \left\{\frac{1}{8}\right\} \left[ 1 + (-3)(kx) + \frac{(-3)(-4)}{2!}(kx)^2 + \frac{(-3)(-4)(-5)}{3!}(kx)^3 + \dots \right]$ $= \left\{\frac{1}{8}\right\} \left[ 1 + (-3)\left(\frac{3x}{2}\right) + \frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2 + \frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3 + \dots \right]$ $= \frac{1}{8} \left[ 1 - \frac{9}{2}x + \frac{27}{2}x^2 - \frac{135}{4}x^3 + \dots \right]$ $= \frac{1}{8} - \frac{9}{16}x + \frac{27}{16}x^2 - \frac{135}{32}x^3 + \dots$	<p><math>\underline{(2)}^{-3}</math> or <math>\underline{\frac{1}{8}}</math> <b>B1</b></p> <p>see notes <b>M1 A1</b></p> <p>See notes below! <b>A1; A1</b></p> <p><b>[5]</b> <b>5</b></p>

Q2	<p>(a) 1.386, 2.291 awrt 1.386, 2.291 <b>B1 B1 (2)</b></p> <p>(b) <math>A \approx \frac{1}{2} \times 0.5( \dots )</math> <b>B1</b>  <math>= \dots (0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545)</math> <b>M1</b>  <math>= 0.25(0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545)</math> ft their (a) <b>A1 ft</b>  <math>= 0.25 \times 29.477 \dots \approx 7.37</math> <b>A1 (4)</b> <span style="float: right;">cao</span></p> <p>(c)(i) <math>\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} \, dx</math> <b>M1 A1</b>  <math>= \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx</math>  <math>= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+C)</math> <b>M1 A1</b></p> <p>(ii) <math>\left[ \frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_1^4 = (8 \ln 4 - 4) - \left( -\frac{1}{4} \right)</math> <b>M1</b>  <math>= 8 \ln 4 - \frac{15}{4}</math>  <math>= 8(2 \ln 2) - \frac{15}{4}</math> <math>\ln 4 = 2 \ln 2</math> seen or implied <b>M1</b>  <math>= \frac{1}{4}(64 \ln 2 - 15)</math> <math>a = 64, b = -15</math> <b>A1 (7)</b></p>	<b>[13]</b>
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Question Number	Scheme	Marks
3. (a)	$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$ $4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$ <p>A method for evaluating one constant</p> $x \rightarrow -\frac{1}{2}, \quad 5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ <p>any one correct constant</p> $x \rightarrow -1, \quad 6 = B(-1)(2) \Rightarrow B = -3$ $x \rightarrow -3, \quad 10 = C(-5)(-2) \Rightarrow C = 1$ <p>all three constants correct</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (4)</p>
(b) (i)	$\int \left( \frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3} \right) dx$ $= \frac{4}{2} \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) + C$ <p>A1 two ln terms correct</p> <p>All three ln terms correct and "+C"; ft constants</p>	<p>M1 A1ft</p> <p>A1ft (3)</p>
(ii)	$\left[ 2 \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) \right]_0^2$ $= (2 \ln 5 - 3 \ln 3 + \ln 5) - (2 \ln 1 - 3 \ln 1 + \ln 3)$ $= 3 \ln 5 - 4 \ln 3$ $= \ln \left( \frac{5^3}{3^4} \right)$ $= \ln \left( \frac{125}{81} \right)$	<p>M1</p> <p>M1</p> <p>A1 (3)</p> <p><b>(10 marks)</b></p>

Question Number	Scheme	Marks
4. (a)	<p>A method of long division gives,</p> $\frac{2(4x^2 + 1)}{(2x + 1)(2x - 1)} \equiv 2 + \frac{4}{(2x + 1)(2x - 1)} \quad A = 2$ $\frac{4}{(2x + 1)(2x - 1)} \equiv \frac{B}{(2x + 1)} + \frac{C}{(2x - 1)}$ $4 \equiv B(2x - 1) + C(2x + 1)$ <p>or their remainder, <math>Dx + E \equiv B(2x - 1) + C(2x + 1)</math></p> <p>Let <math>x = -\frac{1}{2}</math>, <math>4 = -2B \Rightarrow B = -2</math></p> <p>Let <math>x = \frac{1}{2}</math>, <math>4 = 2C \Rightarrow C = 2</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>
4. (b)	<p>Forming any one of these two identities. Can be implied.</p> <p><b>See note below</b></p> <p>either one of <math>B = -2</math> or <math>C = 2</math></p> <p>both <math>B</math> and <math>C</math> correct</p> $\int \frac{2(4x^2 + 1)}{(2x + 1)(2x - 1)} dx = \int 2 - \frac{2}{(2x + 1)} + \frac{2}{(2x - 1)} dx$ $= 2x - \frac{2}{2} \ln(2x + 1) + \frac{2}{2} \ln(2x - 1) (+c)$ <p>Either <math>p \ln(2x + 1)</math> or <math>q \ln(2x - 1)</math></p> <p>or either <math>p \ln 2x + 1</math> or <math>q \ln 2x - 1</math></p> <p><math>A \rightarrow Ax</math></p> <p><math>-\frac{2}{2} \ln(2x + 1) + \frac{2}{2} \ln(2x - 1)</math></p> <p>or <math>-\ln(2x + 1) + \ln(2x - 1)</math></p> <p>See note below.</p> $\int_1^2 \frac{2(4x^2 + 1)}{(2x + 1)(2x - 1)} dx = [2x - \ln(2x + 1) + \ln(2x - 1)]_1^2$ $= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$ $= 2 + \ln 3 + \ln 3 - \ln 5$ $= 2 + \ln\left(\frac{3(3)}{5}\right)$ $= 2 + \ln\left(\frac{9}{5}\right)$ <p>Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)</p> <p>Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for their numerical expression.</p> <p><math>2 + \ln\left(\frac{9}{5}\right)</math></p> <p>Or <math>2 - \ln\left(\frac{5}{9}\right)</math> and k stated as <math>\frac{9}{5}</math>.</p>	<p>[4]</p> <p>M1*</p> <p>B1 ✓</p> <p>A1</p> <p>cso &amp; aef</p> <p>depM1*</p> <p>M1</p> <p>A1</p> <p>[6]</p> <p>10 marks</p>

Question Number	Scheme	Marks
5.	<p>(a) <math>\frac{1}{\sqrt{4-3x}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)}^{-\frac{1}{2}} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}} \left(1 - \frac{3x}{4}\right)^{-\frac{1}{2}}</math></p> $= \frac{1}{2} \left[ 1 + (-\frac{1}{2})(**x); + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (**x)^2 + \dots \right]$ <p><b>with ** <math>\neq 1</math></b></p> $= \frac{1}{2} \left[ 1 + (-\frac{1}{2})(-\frac{3x}{4}) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (-\frac{3x}{4})^2 + \dots \right]$ $= \frac{1}{2} \left[ 1 + \frac{3}{8}x; + \frac{27}{128}x^2 + \dots \right]$ <p>(b) <math>(x+8)\left(\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 + \dots\right)</math></p> $= \frac{1}{2}x + \frac{3}{16}x^2 + \dots$ $+ 4 + \frac{3}{2}x + \frac{27}{32}x^2 + \dots$ $= 4 + 2x; + \frac{33}{32}x^2 + \dots$	<p>B1</p> <p>M1; A1 ft</p> <p>A1 A1 (5)</p> <p>M1</p> <p>M1</p> <p>A1; A1 (4)</p> <p><b>(9 marks)</b></p>

Question Number	Scheme	Marks
6. (a)	$\{y = 0 \Rightarrow\} 1 - 2 \cos x = 0$ $\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	<p>1 - 2 cos x = 0 , seen or implied.  At least one correct value of x. (See notes).  Both <math>\frac{\pi}{3}</math> and <math>\frac{5\pi}{3}</math></p> <p>M1 A1 A1 cso</p>
(b)	$V = \pi \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos x)^2 dx$ $\left\{ \int (1 - 2 \cos x)^2 dx \right\} = \int (1 - 4 \cos x + 4 \cos^2 x) dx$ $= \int 1 - 4 \cos x + 4 \left( \frac{1 + \cos 2x}{2} \right) dx$ $= \int (3 - 4 \cos x + 2 \cos 2x) dx$ $= 3x - 4 \sin x + \frac{2 \sin 2x}{2}$ $V = \{ \pi \} \left( \left( 3 \left( \frac{5\pi}{3} \right) - 4 \sin \left( \frac{5\pi}{3} \right) + \frac{2 \sin \left( \frac{10\pi}{3} \right)}{2} \right) - \left( 3 \left( \frac{\pi}{3} \right) - 4 \sin \left( \frac{\pi}{3} \right) + \frac{2 \sin \left( \frac{2\pi}{3} \right)}{2} \right) \right)$ $= \pi \left( \left( 5\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2} \right) - \left( \pi - 2\sqrt{3} + \frac{\sqrt{3}}{2} \right) \right)$ $= \pi ((18.3060...) - (0.5435...)) = 17.7625\pi = 55.80$ $= \pi (4\pi + 3\sqrt{3}) \text{ or } 4\pi^2 + 3\pi\sqrt{3}$	<p>For <math>\pi \int (1 - 2 \cos x)^2</math>.  Ignore limits and dx</p> <p>cos 2x = 2 cos<sup>2</sup> x - 1  See notes.</p> <p>Attempts <math>\int y^2</math> to give any two of  <math>\pm A \rightarrow \pm Ax</math>, <math>\pm B \cos x \rightarrow \pm B \sin x</math> or  <math>\pm \lambda \cos 2x \rightarrow \pm \mu \sin 2x</math>.  Correct integration.  Applying limits the correct way round. Ignore <math>\pi</math>.</p> <p>Two term exact answer.</p> <p>M1 A1 B1 M1 A1</p>
		<p>[3]</p> <p>[6] 9</p>

Question Number	Scheme	Marks												
7. (a)	<table><tr><td><math>x</math></td><td>0</td><td><math>\frac{\pi}{16}</math></td><td><math>\frac{\pi}{8}</math></td><td><math>\frac{3\pi}{16}</math></td><td><math>\frac{\pi}{4}</math></td></tr><tr><td><math>y</math></td><td>0</td><td>0.44599592 7...</td><td>0.64359425 2...</td><td>0.817421946 ...</td><td>1</td></tr></table>	$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	$y$	0	0.44599592 7...	0.64359425 2...	0.817421946 ...	1	
$x$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$									
$y$	0	0.44599592 7...	0.64359425 2...	0.817421946 ...	1									
	<div></div>	0.446 or awrt 0.44600 B1 awrt 0.64359 B1 awrt 0.81742 B1												
		Outside brackets $\frac{1}{2} \times \frac{\pi}{16}$ or $\frac{\pi}{32}$ B1												
(b)	Area $\approx \frac{1}{2} \times \frac{\pi}{16} \times \{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\}$	For structure of <u>trapezium</u> rule {.....} ; M1 $\sqrt{\quad}$												
		Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$ . A1 $\sqrt{\quad}$												
	$= \frac{\pi}{32} \times 4.81402... = 0.472615308... = \underline{0.4726}$ (4dp)	for seeing <u>0.4726</u> A1 <b>cao</b>												
(c)	Volume $= (\pi) \int_0^{\frac{\pi}{4}} (\sqrt{\tan x})^2 dx = (\pi) \int_0^{\frac{\pi}{4}} \tan x dx$	$\int (\sqrt{\tan x})^2 dx$ or $\int \tan x dx$												
	$= (\pi) [\ln \sec x]_0^{\frac{\pi}{4}}$ or $= (\pi) [-\ln \cos x]_0^{\frac{\pi}{4}}$	Can be implied.												
	$= (\pi) \left[ (\ln \sec \frac{\pi}{4}) - (\ln \sec 0) \right]$	Ignore limits and $(\pi)$												
	or $= (\pi) \left[ (-\ln \cos \frac{\pi}{4}) - (\ln \cos 0) \right]$	$\tan x \rightarrow \ln \sec x$ A1												
	$= \pi \left[ \ln \left( \frac{1}{\sqrt{2}} \right) - \ln \left( \frac{1}{1} \right) \right] = \pi \left[ \ln \sqrt{2} - \ln 1 \right]$	or $\tan x \rightarrow -\ln \cos x$ dM1												
	or $= \pi \left[ -\ln \left( \frac{1}{\sqrt{2}} \right) - \ln(1) \right]$													
	$= \frac{\pi \ln \sqrt{2}}{2}$ or $\frac{\pi \ln \frac{2}{\sqrt{2}}}{2}$ or $\frac{1}{2} \pi \ln 2$ or $\frac{-\pi \ln \left( \frac{1}{\sqrt{2}} \right)}{2}$ or $\frac{\pi}{2} \ln \left( \frac{1}{2} \right)$	$\frac{\pi \ln \sqrt{2}}{2}$ or $\frac{\pi \ln \frac{2}{\sqrt{2}}}{2}$ or $\frac{1}{2} \pi \ln 2$ or $-\pi \ln \left( \frac{1}{\sqrt{2}} \right)$ or $\frac{\pi}{2} \ln \left( \frac{1}{2} \right)$ must be exact. A1 <b>aef</b>												
		[4]												

Question Number	Scheme	Marks
8.	<p>(a) <math>\int (4y+3)^{-\frac{1}{2}} dx = \frac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} (+C)</math>  <math>\left( = \frac{1}{2}(4y+3)^{\frac{1}{2}} + C \right)</math></p> <p>(b) <math>\int \frac{1}{\sqrt{4y+3}} dy = \int \frac{1}{x^2} dx</math>  <math>\int (4y+3)^{-\frac{1}{2}} dy = \int x^{-2} dx</math>  <math>\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} (+C)</math></p> <p>Using <math>(-2, 1.5)</math> <math>\frac{1}{2}(4 \times 1.5 + 3)^{\frac{1}{2}} = -\frac{1}{-2} + C</math>  leading to <math>C = 1</math></p> <p><math>\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + 1</math>  <math>(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}</math>  <math>y = \frac{1}{4} \left( 2 - \frac{2}{x} \right)^2 - \frac{3}{4}</math> or equivalent</p>	<p>M1 A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (6)</p> <p>[8]</p>

### Question 1

This question was generally well answered with about 70% of candidates obtaining all of the 5 marks available.

A minority of candidates were unable to carry out the first step of writing  $(2 + 3x)^{-3}$  as  $\frac{1}{8}\left(1 + \frac{3x}{2}\right)^{-3}$ , with the  $\frac{1}{8}$  outside the brackets usually written incorrectly as either 2 or 1. Many candidates were able to use a correct method for expanding a binomial expression of the form  $(1 + ax)^n$ . A variety of incorrect values of  $a$  were seen, with the most common being either 3,  $\frac{2}{3}$  or 1. Some candidates, having correctly expanded  $\left(1 + \frac{3x}{2}\right)^{-3}$ , forgot to multiply their expansion by  $\frac{1}{8}$ . Errors seen included sign errors, bracketing errors, missing factorials (for example, 2! or 3!) and simplification errors.

### Question 2

Nearly all candidates gained both marks in part (a). As is usual, the main error seen in part (b) was finding the width of the trapezium incorrectly. There were fewer errors in bracketing than had been noted in some recent examinations and nearly all candidates gave the answer to the specified accuracy. The integration by parts in part (c) was well done and the majority of candidates had been well prepared for this topic.

Some failed to simplify  $\int \frac{x^2}{2} \times \frac{1}{x} dx$  to  $\int \frac{x}{2} dx$  and either gave up or produced  $\frac{\frac{1}{3}x^3}{x^2}$ .

In evaluating the definite integral some either overlooked the requirement to give the answer in the form  $\frac{1}{4}(a \ln 2 + b)$  or were unable to use the appropriate rule of logarithms correctly.

### Question 3

Part (a) was well done with the majority choosing to substitute values of  $x$  into an appropriate identity and obtaining the values of  $A$ ,  $B$  and  $C$  correctly. The only error commonly seen was failing to solve  $5 = \frac{5}{4}A$  for  $A$  correctly. Those who formed simultaneous equations in three unknowns tended to be less successful. Any incorrect constants obtained in part (a) were followed through for full marks in part (b)(i). Most candidates obtained logs in part (b)(i). The commonest error was, predictably, giving  $\int \frac{4}{2x+1} dx = 4 \ln(2x+1)$ , although this error was seen less frequently than in some previous examinations. In indefinite integrals, candidates are expected to give a constant of integration but its omission is not penalised repeatedly throughout the paper. In part (b)(ii) most applied the limits correctly although a minority just ignored the lower limit 0. The application of log rules in simplifying the answer was less

successful. Many otherwise completely correct solutions gave  $3\ln 3$  as  $\ln 9$  and some “simplified”  $3\ln 5 - 4\ln 3$  to  $\frac{3}{4}\ln\left(\frac{5}{3}\right)$ .

#### Question 4

This question was well done with many candidates scoring at least eight of the ten marks available.

In part (a), the most popular and successful method was for candidates to multiply both sides of the given identity by  $(2x+1)(2x-1)$  to form a new identity and proceed with “Way 2” as detailed in the mark scheme. A significant proportion of candidates proceeded by using a method (“Way 1”) of long division to find the constant  $A$ . Common errors with this way included algebraic and arithmetic errors in applying long division leading to incorrect remainders; using the quotient instead of the remainder in order to form an identity to find the constants  $B$  and  $C$ ; and using incorrect identities such as  $2(4x^2+1) \equiv B(2x-1) + C(2x+1)$ .

In part (b), the majority of candidates were able to integrate their expression to give an expression of the form  $Ax + p\ln(2x+1) + q\ln(2x-1)$ . Some candidates, however, incorrectly integrated  $\frac{B}{(2x+1)}$  and  $\frac{C}{(2x-1)}$  to give either  $B\ln(2x+1)$  and  $C\ln(2x-1)$  or  $2B\ln(2x+1)$  and  $2C\ln(2x-1)$ . A majority of candidates were able to substitute their limits and use the laws of logarithms to find the given answer. Common errors at this point included either candidates writing  $-\ln(2x+1) + \ln(2x-1)$  as  $\ln(4x^2-1)$ ; or candidates writing  $-\ln 5 + \ln 3$  as either  $\pm \ln 15$  or  $\pm \ln 8$ .

#### Question 5

This question was also generally well tackled with about 50% of candidates obtaining at least 8 of the 9 marks available. A substantial minority of candidates were unable to carry out the first step of

writing  $\frac{1}{\sqrt{4-3x}}$  as  $\frac{1}{2}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ , with the  $\frac{1}{2}$  outside the brackets usually written incorrectly as either

2 or 4. Many candidates were able to use a correct method for expanding a binomial expression of the form  $(1+ax)^n$ . A variety of incorrect values of  $a$  and  $n$ , however, were seen by examiners with the most common being  $a$  as  $\frac{3}{4}$ , 3 and  $-3$  and  $n$  as  $\frac{1}{2}$ ,  $-1$  and  $-2$ . Some candidates, having correctly

expanded  $\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ , forgot to multiply their expansion by  $\frac{1}{2}$ . As expected, sign errors, bracketing errors and simplification errors were also seen in this part. A significant minority of candidates expanded as far as  $x^3$ , and were not penalised on this occasion.

In part (b), most candidates realised that they needed to multiply  $(x+8)$  by their expansion from part (a) although a small minority attempted to divide  $(x+8)$  by their expansion. A surprising number of candidates attempted to expand  $(x+8)$  to obtain a power series. Other candidates omitted the brackets around  $x+8$  although they progressed as if “invisible” brackets were there. The mark scheme allowed candidates to score 2 marks out of 4 even if their answer in (a) was incorrect and many candidates were able to achieve this.

## Question 6

This question was answered well across all abilities.

In Q6(a), most candidates solved  $1 - 2\cos x = 0$  to obtain  $x = \frac{\pi}{3}$ . A number of candidates, however, struggled to find the second value of  $x = \frac{5\pi}{3}$ . A variety of incorrect second values were seen, the most common being  $\frac{2\pi}{3}$ ,  $\frac{4\pi}{3}$  or  $\frac{7\pi}{3}$ .

In Q6(b), the majority of candidates were able to apply volume formula of  $\pi \int y^2 dx$ , although a number of candidates used incorrect formulae such as  $2\pi \int y^2 dx$  or  $\int y^2 dx$  or even  $\int y dx$ . Some candidates incorrectly expanded  $(1 - 2\cos x)^2$  as either  $1 \pm 4\cos^2 2x$  or  $1 - 4\cos x - 2\cos^2 x$  or  $1 - 4\cos x - 4\cos^2 x$ . Others attempted to integrate  $(1 - 2\cos x)^2$  directly to give incorrect expressions such as  $\frac{(1 - 2\cos x)^3}{3(2\sin x)}$ . When the integral included a term in  $\cos^2 x$ , a few candidates integrated this incorrectly to give expressions such as  $\frac{\cos^3 x}{3}$  or  $\frac{\cos^3 x}{-3\sin x}$ . The majority, however, realised the need for using  $\cos 2x = 2\cos^2 x - 1$ . Whilst this double angle formula was generally correctly quoted, this did not always lead to a correct expression for integration as a result of sign or coefficient errors. The integration of an expanded trigonometric expression was generally well done, as was the substitution of the limits found in Q6(a). Candidates are advised to show some evidence of how they have substituted their limits, because this allows some credit to be given if errors occur later in the calculation. Although this question specified an exact answer, decimal answers were occasionally given.

## Question 7

Part (a) was generally well answered as was part (b). In part (a), there were a significant number of candidates, however, who struggled with evaluating  $\tan\left(\frac{\pi}{16}\right)$  and  $\tan\left(\frac{\pi}{8}\right)$  to 5 decimal places and a few other candidates did not change their calculator to radian mode. In part (b), some candidates incorrectly stated the width of each of the trapezia as either 1 or  $\frac{\pi}{20}$ . Nearly all answers were given to 4 decimal places as requested in the question.

Part (c) proved more demanding but it was still pleasing to see many correct solutions. Many candidates who attempted this part were able to integrate  $\tan x$  correctly (given in formula booklet) although this was sometimes erroneously given as  $\sec^2 x$ . There were also a few candidates who attempted to integrate  $\sqrt{\tan x}$ . The substitution of limits caused little difficulty but sometimes a rounded answer was given instead of the required exact answer. Whilst most candidates used  $\pi \int \tan x dx$ ,  $2\pi$  was occasionally seen in place of  $\pi$  and more often  $\pi$  was omitted.

## Question 8

The majority of candidates realised that the answer to part (a) was of the form  $k(4y+3)^{\frac{1}{2}}$  and although the value  $k=2$  was common, most did obtain  $k=\frac{1}{2}$ . In part (b), the majority of candidates knew that they needed to separate the variables, although this was not always done correctly. Those who separated correctly usually were able to integrate  $\frac{1}{x^2}$  correctly, although  $\ln x^2$  was seen from time to time. A significant number of candidates did not use a constant of integration and could gain no further marks in the question. It is disappointing to report that many otherwise correct solutions were spoiled by elementary algebraic errors. Many candidates obtained a correct expression, for example,  $(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}$  but were unable to make  $y$  the subject of the formula correctly. For examples,  $4y+3 = 4 + \frac{4}{x^2}$  and, even,  $4y+3 = \sqrt{2 - \frac{2}{x}}$  were often seen.

## Statistics for C4 Practice Paper Bronze Level B2

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	5	5	88	4.42	4.90	4.71	4.49	4.22	3.72	3.33	2.46
2	13		77	10.06		11.88	9.64	8.27	6.96	4.98	3.39
3	10		77	7.66		9.01	7.97	7.00	5.97	4.76	3.04
4	10		73	7.31		8.92	7.56	6.44	5.17	3.96	2.28
5	9		76	6.80		8.12	7.16	6.26	5.13	3.70	1.89
6	9	9	67	6.02	8.47	7.04	5.84	4.45	3.41	2.76	1.38
7	11		75	8.23		9.84	8.60	7.43	6.09	4.88	3.11
8	8		52	4.19	7.32	5.79	4.04	2.50	1.48	0.91	0.52
	75		73	54.69		65.31	55.30	46.57	37.93	29.28	18.07